Exercise 9B

1 a $\int x^3 dx = \frac{x^4}{4} + c$

b $\int x^7 dx = \frac{x^8}{8} + c$

c $\int 3x^{-4} dx = \frac{3x^{-3}}{-3} + c = -x^{-3} + c$

d $\int 5x^2 dx = \frac{5x^3}{3} + c$

2 a $\int (x^4 + 2x^3)\, dx = \frac{5x^5}{5} + \frac{2x^4}{4} + c$

$= x^5 + \frac{x^4}{2} + c$

b $\int (2x^3 - x^2 + 5x)\, dx = \frac{2x^4}{4} - \frac{x^3}{3} + \frac{5x^2}{2} + c$

$= \frac{x^4}{2} - \frac{x^3}{3} + \frac{5x^2}{2} + c$

c $\int (5x^{\frac{3}{2}} - 3x^2)\, dx = \frac{5x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{3x^3}{3} + c$

$= 2x^{\frac{5}{2}} - x^3 + c$

3 a $\int (4x^{-2} + 3x^{-\frac{1}{2}})\, dx = \frac{4x^{-1}}{-1} + \frac{3x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= -4x^{-1} + 6x^{\frac{1}{2}} + c$

b $\int (6x^{-2} - x^{-\frac{1}{2}})\, dx = \frac{6x^{-2}}{-1} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= -6x^{-2} - 2x^{\frac{1}{2}} + c$

c $\int (2x^{-\frac{3}{2}} + x^2 - x^{-\frac{1}{2}})\, dx$

$= \frac{2x^{-\frac{1}{2}}}{-\frac{1}{2}} + \frac{x^3}{3} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= -4x^{-\frac{1}{2}} + \frac{x^3}{3} - 2x^{\frac{1}{2}} + c$

4 a $\int (4x^3 - 3x^{-4} + 2)\, dx = \frac{4x^4}{4} - \frac{3x^{-3}}{-3} + 2x + c$

$= x^4 + x^{-3} + 2x + c$

b $\int (x + x^{-\frac{1}{2}} + x^{-\frac{3}{2}})\, dx = \frac{x^2}{2} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + c$

$= \frac{x^2}{2} + 2x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} + c$

c $\int (px^4 + 2q + 3x^{-2})\, dx$

$= \frac{px^5}{5} + 2qx + 3\frac{x^{-1}}{-1} + c$

$= \frac{px^5}{5} + 2q - \frac{3}{x} + c$

5 a $\int (3t^2 - t^{-2})\, dt = \frac{3t^3}{3} - \frac{t^{-1}}{-1} + c$

$= t^3 + t^{-1} + c$

b $\int (2t^2 - 3t^{-\frac{3}{2}} + 1)\, dt = \frac{2t^3}{3} - \frac{3t^{-\frac{1}{2}}}{-\frac{1}{2}} + t + c$

$= \frac{2t^3}{3} + 6t^{-\frac{1}{2}} + t + c$

c $\int (pt^3 + q^2 + pr^3)\, dt$

$= \frac{pt^4}{4} + q^2 t + pr^3 t + c$

6 a $\int \frac{(2x^2 + 3)}{x^2}\, dx = \int \left(\frac{2x^2}{x^2} + \frac{3}{x^2}\right) dx$

$= \int \left(2 + 3x^{-2}\right) dx$

$= 2x + \frac{3x^{-1}}{-1} + c$

$= 2x - \frac{3}{x} + c$

6 b $\int (2x+3)^2 \,dx = \int (4x^2+12x+9)\,dx$

$$= \frac{4x^3}{3} + 6x^2 + 9x + c$$

c $\int (2x+3)\sqrt{x}\,dx = \int (2x+3)x^{\frac{1}{2}}\,dx$

$$= \int \left(2x^{\frac{3}{2}} + 3x^{\frac{1}{2}}\right) dx$$

$$= \frac{2x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= \frac{4x^{\frac{5}{2}}}{5} + 2x^{\frac{3}{2}} + c$$

7 a $\int \left(x + \frac{1}{x}\right)^2 dx = \int \left(x^2 + 2 + \frac{1}{x^2}\right) dx$

$$= \int \left(x^2 + 2 + x^{-2}\right) dx$$

$$= \frac{x^3}{3} + 2x + \frac{x^{-1}}{-1} + c$$

$$= \frac{x^3}{3} + 2x - \frac{1}{x} + c$$

b $\int \left(\sqrt{x}+2\right)^2 dx = \int \left(x + 4\sqrt{x} + 4\right) dx$

$$= \int \left(x + 4x^{\frac{1}{2}} + 4\right) dx$$

$$= \frac{x^2}{2} + \frac{4x^{\frac{3}{2}}}{\frac{3}{2}} + 4x + c$$

$$= \frac{x^2}{2} + \frac{8x^{\frac{3}{2}}}{3} + 4x + c$$

c $\int \left(\frac{1}{\sqrt{x}} + 2\sqrt{x}\right) dx = \int \left(x^{-\frac{1}{2}} + 2x^{\frac{1}{2}}\right) dx$

$$= \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{2x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= 2x^{\frac{1}{2}} + \frac{4x^{\frac{3}{2}}}{3} + c$$

8 a $\int \left(x^{\frac{2}{3}} + \frac{4}{x^3}\right) dx = \int \left(x^{\frac{2}{3}} + 4x^{-3}\right) dx$

$$= \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + \frac{4x^{-2}}{-2} + c$$

$$= \frac{3x^{\frac{5}{3}}}{5} - \frac{2}{x^2} + c$$

b $\int \left(\frac{2+x}{x^3} + 3\right) dx = \int \left(2x^{-3} + x^{-2} + 3\right) dx$

$$= \frac{2x^{-2}}{-2} + \frac{x^{-1}}{-1} + 3x + c$$

$$= -\frac{1}{x^2} - \frac{1}{x} + 3x + c$$

c $\int \left(x^2+3\right)(x-1)\,dx = \int \left(x^3 - x^2 + 3x - 3\right) dx$

$$= \frac{x^4}{4} - \frac{x^3}{3} + \frac{3x^2}{2} - 3x + c$$

d $\int \frac{(2x+1)^2}{\sqrt{x}} dx = \int \left(\frac{4x^2+4x+1}{x^{\frac{1}{2}}}\right) dx$

$$= \int \left(4x^{\frac{3}{2}} + 4x^{\frac{1}{2}} + x^{\frac{-1}{2}}\right) dx$$

$$= \frac{8x^{\frac{5}{2}}}{5} + \frac{8x^{\frac{3}{2}}}{3} + 2x^{\frac{1}{2}} + c$$

e $\int \left(3 + \frac{\sqrt{x} + 6x^3}{x}\right) dx = \int \left(3 + x^{\frac{-1}{2}} + 6x^2\right) dx$

$$= 3x + 2x^{\frac{1}{2}} + 2x^3 + c$$

f $\int \sqrt{x}\left(\sqrt{x}+3\right)^2 dx = \int x^{\frac{1}{2}}\left(x + 6x^{\frac{1}{2}} + 9\right) dx$

$$= \int \left(x^{\frac{3}{2}} + 6x + 9x^{\frac{1}{2}}\right) dx$$

$$= \frac{2x^{\frac{5}{2}}}{5} + 3x^2 + 6x^{\frac{3}{2}} + c$$

9 a $\int\left(\frac{A}{x^2}-3\right)\mathrm{d}x = \int(Ax^{-2}-3)\,\mathrm{d}x$

$$= \frac{Ax^{-1}}{-1} - 3x + c$$

$$= -\frac{A}{x} - 3x + c$$

9 b $\int\left(\sqrt{P}x+\frac{2}{x^3}\right)\mathrm{d}x = \int\left(\sqrt{P}x^{\frac{1}{2}}+2x^{-3}\right)\mathrm{d}x$

$$= \frac{\sqrt{P}x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{2x^{-2}}{-2} + c$$

$$= \frac{2\sqrt{P}x^{\frac{3}{2}}}{3} - \frac{1}{x^2} + c$$

9 c $\int\left(\frac{p}{x^2}+q\sqrt{x}+r\right)\mathrm{d}x$

$$= \frac{px^{-1}}{-1} + \frac{qx^{\frac{3}{2}}}{\frac{3}{2}} + rx + c$$

$$= -\frac{p}{x} + \frac{2qx^{\frac{3}{2}}}{3} + rx + c$$

10 $\int\left(\frac{6}{x^2}+4\sqrt{x}-3x+2\right)\mathrm{d}x$

$$= \int(6x^{-2}+4x^{\frac{1}{2}}-3x+2)\,\mathrm{d}x$$

$$= \frac{6x^{-1}}{-1} + \frac{4x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{3x^2}{2} + 2x + c$$

$$= -\frac{6}{x} + \frac{8x^{\frac{3}{2}}}{3} - \frac{3x^2}{2} + 2x + c$$

11 $\int\left(8x^3+6x-\frac{3}{\sqrt{x}}\right)\mathrm{d}x$

$$= \int(8x^3+6x-3x^{-\frac{1}{2}})\,\mathrm{d}x$$

$$= \frac{8x^4}{4} + \frac{6x^2}{2} - \frac{3x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= 2x^4 + 3x^2 - 6\sqrt{x} + c$$

12 a $(2+5\sqrt{x})^2 = (2+5\sqrt{x})(2+5\sqrt{x})$

$$= 4+10\sqrt{x}+10\sqrt{x}+25x$$

$$= 4+20\sqrt{x}+25x$$

So $k = 20$

b $\int(4+20\sqrt{x}+25x)\,\mathrm{d}x$

$$= \int(4+20x^{\frac{1}{2}}+25x)\,\mathrm{d}x$$

$$= 4x + \frac{20x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{25x^2}{2} + c$$

$$= 4x + \frac{40x^{\frac{3}{2}}}{3} + \frac{25x^2}{2} + c$$

13 $\int\left(3x^5-\frac{4}{\sqrt{x}}\right)\mathrm{d}x = \int(3x^5-4x^{-\frac{1}{2}})\,\mathrm{d}x$

$$= \frac{3x^6}{6} - \frac{4x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= \frac{x^6}{2} - 8\sqrt{x} + c$$

14 $\int\left(\frac{p}{2x^2}+pq\right)\mathrm{d}x = \int\left(\frac{px^{-2}}{2}+pq\right)\mathrm{d}x$

$$= \frac{px^{-1}}{2\times -1} + pqx + c$$

$$= -\frac{p}{2x} + pqx + c$$

$$= \frac{2}{x} + 10x + c$$

$-\frac{p}{2} = 2$ and $pq = 10$

$p = -4$ and $q = -2.5$